ECE 3830 Review Problems   Fall 2006

P1 Evaluate the following integrals

a) $\int_{-\infty}^{+\infty} \sin[3(t-1)]\, \delta(2t+4)\, dt$

b) $\int_{-\infty}^{+\infty} t^2 u(t-2)\, \delta(2-t)\, dt$

P2 Given a system input $x(t)$ and a system response funct $h(t)$, find the output $y(t)$

a) $x(t) = e^{-t} u(t-1) \qquad h(t) = e^{-2t} u(t-2)$

b) $x(t) = 2u(1-t) \qquad h(t) = e^{-at} u(2-t)$

✓ **P3** Find the Fourier coefficients, both exponential and trigonometric ones for the function in Fig 4.10(a) – (in the textbook

✓ **P4** Hw #2, Problem 4

✓ **P5** Hw #2, Problem 5

✓ **P6** Problem 5.1 (textbook)

✓ **P7** Problem 5.8 (textbook)

✓ **P8** Problem 5.11 (textbook)

✓ **P9** Problem 5.12 (textbook)

✓ **P10** Problem 5.16 (textbook)

P11 Problem 5.18 (textbook)

P12 Problem 6.13 (textbook)

P13 Problem 6.15 (textbook)

✓ P14 Problem 10.9 (textbook)

P15 Problem 10.19 (a) (textbook)
Also, solve these difference equations using Z-transform

P16 Problem 11.10 (a)

ECE 3830 Review Problem Solution

P1

$$\int_{-\infty}^{+\infty} f(t)\,\delta(at+b)\,dt$$

$$= \frac{1}{|a|} f(t) \Big|_{t=\frac{-b}{a}}$$

a) $$\int_{-\infty}^{+\infty} \sin[3(t-1)]\,\delta(2t+4)\,dt$$

$$= \frac{1}{|2|} \sin[3(t-1)] \Big|_{t=-2}$$

$$= \frac{1}{2}\sin(-9) = -\frac{1}{2}\sin(9)$$

b) $$\int_{-\infty}^{+\infty} t^2 u(t-2)\,\delta(2-t)\,dt$$

$$= \frac{1}{|-1|} t^2 u(t-2) \Big|_{t=2}$$

$$= 4\,u(2-2) = 4$$

P2 convolution

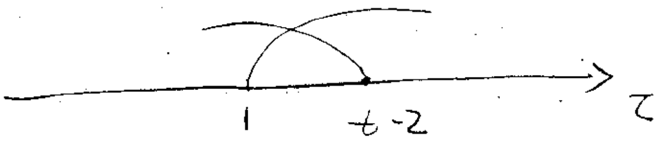
a) $$x(t) * h(t) = \int_{-\infty}^{+\infty} x(\tau)\,h(t-\tau)\,d\tau$$

$$= \int_{-\infty}^{+\infty} e^{-\tau} u(\tau-1)\, e^{-2(t-\tau)} u(t-\tau-2)\,d\tau$$

$$= e^{-2t} \int_{-\infty}^{+\infty} e^{-\tau} e^{2\tau} u(\tau-1)\,u(t-\tau-2)\,d\tau$$

$$u(\tau-1) = \begin{cases} 1 & \tau-1 \geq 0 \quad \tau \geq 1 \\ 0 & \text{else} \end{cases}$$

$$u(t-\tau-2) = \begin{cases} 1 & t-\tau-2 \geq 0 \quad \tau \leq t-2 \\ 0 & \text{else} \end{cases}$$



CASE A $\quad t-2 \geq 1 \rightarrow t-3 \geq 0$

$$x(t) * h(t) = e^{-2t} \cdot \int_{1}^{t-2} e^{\tau} d\tau$$

$$= e^{-2t}\left[e^{t-2} - e^{1}\right]$$

CASE B [illegible]

$$x(t) * h(t) = 0$$

therefore $\quad x(t) * h(t) = e^{-2t}\left[e^{t-2} - e\right] u(t-3)$

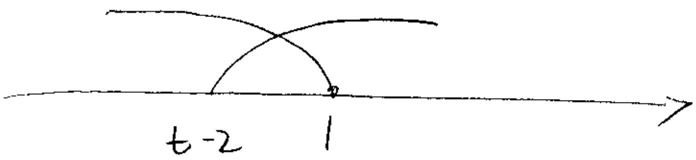
b) $x(t) * h(t)$

$$= \int_{-\infty}^{+\infty} 2u(1-\tau)\, e^{-a(t-\tau)} u(2-(t-\tau))\, d\tau$$

$$= 2e^{-at} \int_{-\infty}^{+\infty} u(1-\tau)\, u(2-t+\tau)\, e^{a\tau} d\tau$$

$$u(1-\tau) = \begin{cases} 1 & 1-\tau \geq 0 \quad \tau \leq 1 \\ 0 & \text{else} \end{cases}$$

$$u(2-t+\tau) = \begin{cases} 1 & 2-t+\tau \geq 0 \Rightarrow \tau \geq t-2 \\ 0 & 0 \end{cases}$$



CASE A $\quad t-2 \le 1 \quad\quad 1-t+2 \geq 0$

$$x(t) * h(t) = 2e^{-at}\int_{t-2}^{1} e^{a\tau}\,d\tau$$
$$= \frac{1}{a} 2e^{-at}\left[e^{a} - e^{a(t-2)}\right]$$
$$= \frac{2}{a}\left[e^{-at+a} - e^{-2a}\right]$$

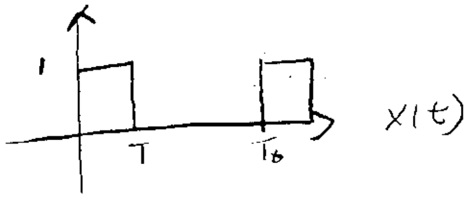
CASE B $\quad t-2 > 1$

$$x(t) * h(t) = 0$$

Therefore
$$x(t) * h(t) = \frac{2}{a}\left[e^{-at+a} - e^{-2a}\right] u(3-t)$$

Problem 1

Square wave



$$x(t) = \sum_{n=-\infty}^{+\infty} c_n e^{+jn\omega_0 t}$$

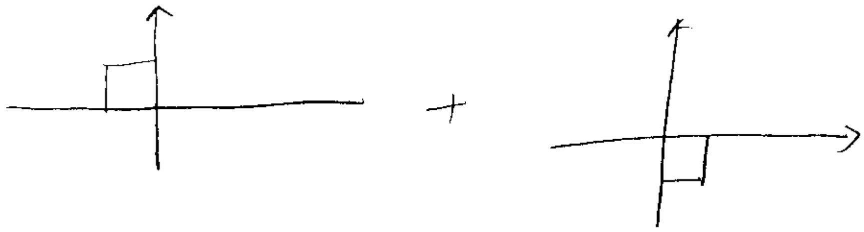
$$c_n = \frac{1}{T_0} \int_0^{T_0} x(t) e^{-jn\omega_0 t}\, dt$$

$$= \frac{1}{T_0} \int_0^{T} e^{-jn\omega_0 t}\, dt$$

$$= \frac{1}{T_0} \frac{1}{-jn\omega_0} e^{-jn\omega_0 t}\Big|_0^T$$

$$= \frac{1}{jn2\pi}\left[1 - e^{-jn\omega_0 T}\right]$$

a) $x_a(t) = 3[x(t+1) - x(t)]$



$T = 1 \quad T_0 = 4 \quad \omega_0 = \frac{\pi}{2}$

$x(t+1)$: using time shifting property

$$d_n = c_n e^{-jn\omega_0(-1)}$$

$$x_a(t) = 3 c_n \left[e^{+jn\omega_0} - 1 \right]$$

$$= 3 \frac{1}{jn2\pi}\left[1 - e^{-jn\frac{\pi}{2}}\right]\left[e^{jn\frac{\pi}{2}} - 1\right]$$

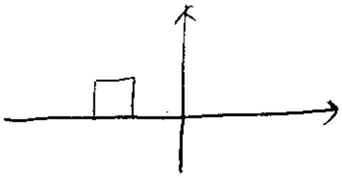
$$= \frac{3}{jn2\pi}\left[e^{jn\frac{\pi}{2}} - 1 - 1 - e^{-jn\frac{\pi}{2}}\right]$$

$$= \frac{j3}{n\pi}\left[1 - \cos\frac{n\pi}{2}\right]$$

$C_0 = 0$, function is odd.

Then Find $\{A_n \ B_n\}$ $A_n = [C_n + C_{-n}]$ $B_n = \dfrac{C_n - C_{-n}}{j}$

b)

$X_b(t) =$  + 

$x(t+2) \quad + \; 2x(t+1)$

$$d_n = C_n\left[e^{-jn\omega_0(-2)} + 2e^{-jn\omega_0(-1)}\right], \quad \omega_0 = \frac{\pi}{2}$$

$$= \frac{1}{jn2\pi}\left[1 - e^{-jn\frac{\pi}{2}}\right]\left[e^{+jn\pi} + 2e^{jn\frac{\pi}{2}}\right]$$

Then Find $\{A_n \ B_n\}$

c)

$T_0 = 2 \quad \omega_0 = \pi$

$$C_n = \frac{1}{T_0}\int_0^{T_0} x(t)e^{-jn\omega_0 t}\,dt$$

$$= \frac{1}{2}\int_0^1 2t\, e^{-jn\pi t}\,dt$$

$$= \int_0^1 t\, e^{-jn\pi t}\,dt$$

integration by parts

$$= \frac{1}{-jn\pi}\int_0^1 t\, d\, e^{-jn\pi t}$$

$$= \frac{1}{-jn\pi}\left[t e^{-jn\pi t}\Big|_0^1 - \int_0^1 e^{-jn\pi t}\,dt\right]$$

$$= \frac{1}{-jn\pi}\left[1 \cdot e^{-jn\pi} - 0 - \int_0^1 e^{-jn\pi t}\, dt\right]$$

$$= \frac{1}{-jn\pi}\left[e^{-jn\pi} - \frac{1}{-jn\pi} e^{-jn\pi t}\Big|_0^1\right]$$

$$= \frac{1}{-jn\pi}\left(e^{-jn\pi} + \frac{1}{jn\pi}\left[e^{-jn\pi} - 1\right]\right)$$

$$= -e^{-jn\pi}\left[\frac{1}{jn\pi} - \left(\frac{1}{jn\pi}\right)^2\right] - \frac{1}{n^2\pi^2}$$

$$C_0 = \frac{1}{T_0}\int_0^{T_0} f(t)\, dt = \frac{1}{2}$$

Then Find $\underline{A_n, B_n}$

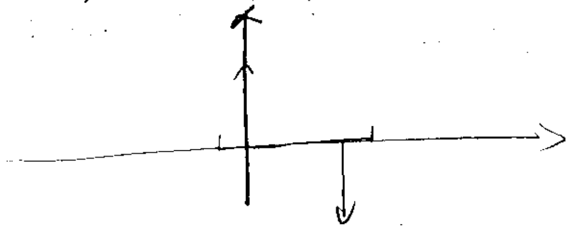
$\boxed{d)}$

$$x_d(t) = -x_c(t) + 2$$

$$d_n = -C_n \qquad n \neq 1$$

$$d_0 = C_0 + 2$$

Problem 2 4.14



$T_0 = 0.2 \quad w_0 = \frac{2\pi}{0.2} = 10\pi$

$$C_n = \frac{1}{T_0}\int_{-0.05}^{0.15} x(t)\, e^{-jnw_0t}\, dt$$

$$= \frac{1}{T_0}\int_{-0.05}^{0.15} \left[\delta(t) + \delta(t-0.1)\right] e^{-jnw_0t}\, dt$$

$$= \frac{1}{0.2}\left[e^{-jn10\pi t}\Big|_{t=0} + e^{-jn10t}\Big|_{t=0.1}\right]$$

$$= 5\left[1 + e^{-jn}\right]$$

$C_0 = 10$

Problem 3

using trigonometric Fourier series

$x(t)$ even $b_n = 0$

$T_0 = 2\pi \quad w_0 = 1$

$$C_n = \frac{2}{2\pi}\int_{-\pi}^{\pi} t^2 \cos nt\, dt$$

$$= \frac{2}{2\pi}\frac{1}{n}\int_{-\pi}^{\pi} t^2 \, d(\sin nt)$$

$$= \frac{2}{2\pi n}\left[t^2 \sin nt \Big|_{-\pi}^{\pi} - \int_{-\pi}^{\pi} \sin nt \cdot 2t \, dt\right]$$

$$= \frac{2}{2\pi n}\left[0 - 2\int_{-\pi}^{\pi} \sin nt \; t \, dt\right]$$

$$= \frac{2}{\pi n} \quad \frac{1}{n} \int_{-\pi}^{\pi} t \, d(\cos nt)$$

$$= \frac{2}{\pi n^2}\left[\cos nt \cdot t \Big|_{-\pi}^{\pi} - \int_{-\pi}^{\pi} \cos nt \, dt\right]$$

$$= \frac{2}{\pi n^2}\left[\pi \cos n\pi + \pi \cos n\pi - 0\right]$$

$$= \frac{4}{n^2} \cos n\pi$$

$$c_0 = \frac{\pi^2}{3}$$

$$c_1 = -4$$

$$c_2 = 1 \qquad c_3 = -\frac{4}{9} \quad \cdots$$

Let $t=0$

$$0 = x(0) = \frac{\pi^2}{3} - 4\left(1 - \frac{1}{4} + \frac{1}{9} - \quad\right)$$

$$\Rightarrow \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} = \frac{\pi^2}{12}$$

$$= \frac{4}{2\pi} \int_0^{\pi} t^2 \cos nt \, dt$$

$$= \frac{2}{\pi} \frac{1}{n} \int_0^{\pi} t^2 \, d(\sin nt)$$

$$= \frac{2}{\pi n} \left[t^2 \sin nt \Big|_0^{\pi} - \int_0^{\pi} \sin nt \cdot 2t \, dt \right]$$

$$= \frac{2}{\pi n} \cdot 2 \; (-1)(-1) \frac{1}{n} \int_0^{\pi} t \; d(\cos nt)$$

$$= \frac{2}{\pi n^2} \left[t \cos nt \Big|_0^{\pi} - \int_0^{\pi} \cos nt \, dt \right]$$

$$= \frac{1}{\pi n^2}$$

Problem 4

$$x(t) = 2 + \frac{1}{2}\cos\left(t + \frac{\pi}{4}\right) + 2\cos(3t) - 2\sin\left(4t + \frac{\pi}{6}\right)$$

$$= 2 + \frac{1}{2}\left(e^{jt} e^{j\frac{\pi}{4}} + e^{jt} e^{j\frac{\pi}{4}}\right)\frac{1}{2}$$

$$+ 2\left(e^{j3t} + e^{-j3t}\right)\frac{1}{2}$$

$$- 2 \cdot \frac{1}{2j}\left(e^{j4t} e^{j\frac{\pi}{6}} - e^{-j4t} e^{-j\frac{\pi}{6}}\right)$$

$c_0 = 2$

$c_1 = c_{-1} = \frac{1}{4} e^{j\frac{\pi}{4}}$

$c_3 = c_{-3} = \frac{1}{4}$

$c_4 = j e^{j\frac{\pi}{6}} \qquad c_{-4} = -j e^{-j\frac{\pi}{6}}$

other, $c_n = 0$.

Problem 5

a) $x(t) = \sum_{n=-\infty}^{+\infty} C_n e^{jn\omega_0 t}$

$C_n e^{jn\omega_0 t} + C_{-n} e^{-jn\omega_0 t}$

$= \frac{1}{n^2+1} e^{jn\frac{\pi}{3}} e^{jn\omega_0 t} + \frac{1}{n^2+1} e^{-jn\frac{\pi}{3}} e^{-jn\omega_0 t}$

$= \frac{1}{n^2+1} \cos(n\omega_0 t + n\frac{\pi}{3})$

So $x(t)$ is real

b) $C_0 = \frac{1}{1} e^{j0\frac{\pi}{3}} = 1$ → average value

c) Need to find transform function.

$H(\omega) = \frac{1}{1+j\omega RC}$

$d_n = C_n \, H(\omega)\big|_{\omega = jn\omega_0}$

$= \frac{1}{n^2+1} e^{jn\frac{\pi}{3}} \frac{1}{1+jn\omega_0 \cdot 1}$ $\quad |d_n| = |C_n| \, |H(jn\omega_0)|$

$= \dots\dots$

d) The high-frequency terms are suppressed.

end

150 HW#3 5.1 (40) 5.8 (30) 5.11 (20) 5.12 (20) 5.16 (a-e) (25) 5.23 (15)

1 a) $x(t) = 2[u(t) - u(t-4)]$

$$X(\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt$$

$$= \int_{-\infty}^{+\infty} 2[u(t) - u(t-4)] e^{-j\omega t} dt$$

$$= \int_{0}^{4} 2 e^{-j\omega t} dt$$

$$= \frac{2}{j\omega}[1 - e^{-4j\omega}]$$

b) $x(t) = e^{-3t}[u(t) - u(t-4)]$

$$X(\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt$$

$$= \int_{0}^{4} e^{-3t} e^{-j\omega t} dt$$

$$= \int_{0}^{4} e^{-(3+j\omega)t} dt$$

$$= \frac{1}{3+j\omega}[1 - e^{-4(3+j\omega)}]$$

c) $x(t) = 2t[u(t) - u(t-4)]$

$$X(\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt$$

$$= 2\int_{0}^{4} t \, e^{-j\omega t} dt$$

integration by parts

$$= 2\int_0^4 t \, d\, e^{-j\omega t} \cdot \frac{1}{-j\omega}$$

$$= \frac{2}{-j\omega}\left[e^{-j\omega t}\, t \Big|_0^4 - \int_0^4 e^{-j\omega t} dt \right]$$

$$= \frac{2}{-j\omega}\left[4e^{-j\omega 4} - 0 - \frac{1}{j\omega}(1 - e^{-j\omega 4}) \right]$$

$$= \frac{2}{w^2}\left[j\omega\, e^{-j\omega 4} - j\omega - e^{-j\omega 4} \right]$$

d) $x(t) = \cos 4\pi t \, [u(t+2) - u(t-2)]$

$$X(\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt$$

$$= \int_{-2}^{2} \cos 4\pi t \; e^{-j\omega t} \, dt$$

$$= \frac{1}{2}\int_{-2}^{2} e^{j4\pi t} e^{-j\omega t} dt + \frac{1}{2}\int_{-2}^{2} e^{-j4\pi t} e^{-j\omega t}$$

$$= \frac{1}{2}\int_{-2}^{2} e^{j(4\pi-\omega)t} dt + \frac{1}{2}\int_{-2}^{2} e^{-j(4\pi+\omega)t}$$

$$= \frac{1}{2}\,\frac{1}{j(4\pi-\omega)}\left[e^{j(4\pi-\omega)2} - e^{j(4\pi-\omega)(-2)} \right]$$

$$+ \frac{1}{2}\,\frac{1}{-j(4\pi+\omega)}\left[e^{-j(4\pi+\omega)2} - e^{-j(4\pi+\omega)(-2)} \right]$$

$$= \frac{1}{j(4\pi-\omega)} \sin 2(4\pi-\omega)\, j + \frac{1}{-j(4\pi+\omega)} \sin 2(4\pi+\omega)(-1)$$

$$= \frac{\sin 2(4\pi-\omega)}{(4\pi-\omega)} + \frac{\sin 2(4\pi+\omega)}{(4\pi+\omega)}$$

5.8 $e^{-|t|} \leftrightarrow \frac{2}{\omega^2+1}$

(a) $\frac{d}{dt} e^{-|t|} \leftrightarrow \frac{2}{\omega^2+1} \cdot j\omega$

(b) $\frac{1}{2\pi(t^2+1)} \leftrightarrow \frac{1}{2} e^{-|\omega|}$

dual property $f(t) \leftrightarrow F(\omega)$

$F(t) \leftrightarrow 2\pi f(-\omega)$

$e^{-|t|} \leftrightarrow \frac{2}{\omega^2+1}$

$\frac{2}{t^2+1} \leftrightarrow 2\pi e^{-|-\omega|} = 2\pi e^{-|\omega|}$

(c) $\frac{4\cos(2t)}{t^2+1}$

$\frac{1}{t^2+1} \leftrightarrow 2\pi e^{-|\omega|}$

$f(t) \leftrightarrow F(\omega)$

$f(\omega) \cos \omega_0 t \leftrightarrow \frac{1}{2}[F(\omega+\omega_0) + F(\omega-\omega_0)]$

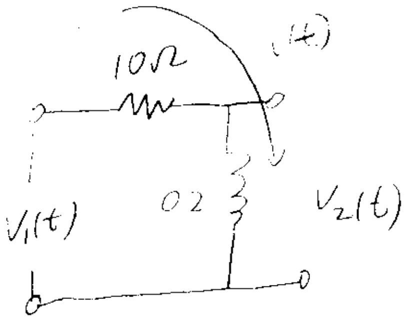
$\frac{4\cos(2t)}{t^2+1}$

$\leftrightarrow 4 \cdot \frac{1}{2}\left[2\pi e^{-|\omega+\omega_0|} + 2\pi e^{-|\omega-\omega_0|}\right]$

$= 4\pi\left[e^{-|\omega+\omega_0|} + e^{-|\omega-\omega_0|}\right]$

5.12

(a)



$V_R(t) = i(t)\, R$

$V_R(\omega) = I(\omega)\, R = 10\, I(\omega)$ ①

$v_2(t) = L \frac{d i(t)}{dt}$

$V_2(\omega) = L\, j\omega\, I(\omega) = 0.2\, j\omega\, I(\omega)$ ②

$V_1(t) = V_R(t) + V_2(t)$

$V_1(\omega) = V_R(\omega) + V_2(\omega)$

① ②: $\frac{V_R(\omega)}{10} = I(\omega)$

$\frac{V_2(\omega)}{0.2 j\omega} = I(\omega)$

$V_R(\omega) = 10\, I(\omega) = 10 \frac{V_2(\omega)}{0.2 j\omega}$

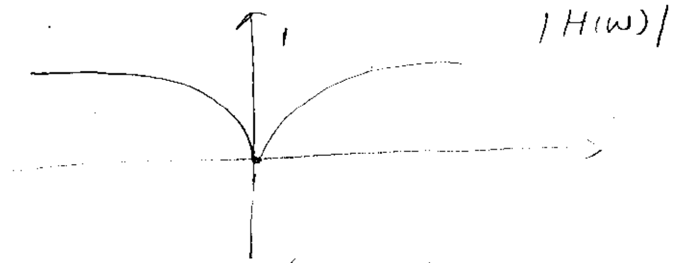
$= 50 \frac{V_2(\omega)}{j\omega} + V_2(\omega)$

$= V_2(\omega)\left[1 + \frac{50}{j\omega}\right]$

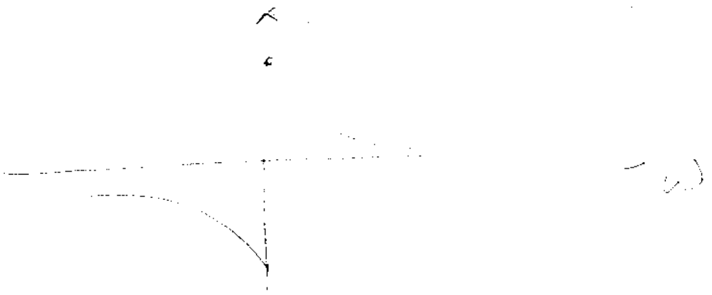
$V_2(\omega) = H(\omega)\, V_1(\omega)$

$$H(\omega) = \frac{V_2(\omega)}{V_1(\omega)} = \frac{1}{1 + 50/j\omega} = \frac{j\omega}{j\omega + 50}$$

b) $$|H(\omega)| = \frac{|j\omega|}{|j\omega + 50|} = \frac{|\omega|}{\sqrt{50^2 + \omega^2}}$$



$$\measuredangle H(\omega) = \measuredangle j\omega - \measuredangle (j\omega + 50)$$
$$= \frac{\pi}{2} - \tan^{-1}\left(\frac{\omega}{50}\right)$$



(c) $e^{-at}u(t) \leftrightarrow \frac{1}{a+j\omega}$

$$H(\omega) = \frac{j\omega}{50+j\omega} = 1 - \frac{50}{50+j\omega}$$

$$h(t) = \delta(t) - 50e^{-50t}u(t)$$

5.6 $g(t) \leftrightarrow G(\omega)$

(a) $g(4t) \leftrightarrow \frac{1}{|4|}G\left(\frac{\omega}{4}\right) = \frac{1}{4}\,\frac{j\frac{\omega}{4}}{-\frac{\omega^2}{16} + 7j\frac{\omega}{4} + 6}$

(b) $g(6t-12) = g(6(t-2))$

$$g(6t) \leftrightarrow \frac{1}{6} G\left(\frac{\omega}{6}\right)$$

$$g(6(t-2)) \leftrightarrow \frac{1}{6} G\left(\frac{\omega}{6}\right) \cdot e^{-j2\omega}$$

$$= \frac{1}{6} \frac{j\frac{\omega}{6}}{-\left(\frac{\omega}{6}\right)^2 + 7j\frac{\omega}{6} + 6} e^{-j2\omega}$$

(c) $$\frac{d\,g(t)}{dt} \leftrightarrow j\omega G(\omega) = \frac{-\omega^2}{-\omega^2 + 7j\omega + 6}$$

(d) $$g(-t) \leftrightarrow \frac{1}{|-1|} G(-\omega) = \frac{-j\omega}{-\omega^2 + 7(-\omega)j + 6}$$

(e) $$e^{-j200t} g(t) \leftrightarrow G(\omega + 200)$$

$$= \frac{j(\omega+200)}{-(\omega+200)^2 + 7j(\omega+200) + 6}$$

10.9 a)

$x[n] = a^{-3n} u[1-n] \qquad h[n] = b^{n} u[2-n]$

$y[n] = \sum_{k=-\infty}^{+\infty} x[k]\, h[n-k]$

$= \sum_{k=-\infty}^{+\infty} a^{-3k} u[1-k]\, b^{n-k} u[2-(n-k)]$

$= b^{n} \sum_{k=-\infty}^{+\infty} a^{-3k} b^{-k} u[1-k]\, u[2-n+k]$

$u[1-k] = \begin{cases} 1 & k \le 1 \\ 0 & k > 1 \end{cases}$

$u[2-n+k] = \begin{cases} 1 & 2-n+k \geq 0 \Rightarrow k \geq n-2 \\ 0 & k < n-2 \end{cases}$

CASE A $\quad n-2 \geqslant 1 \quad \Rightarrow n-3 \geqslant 0$

$y[n] = b^{n} \sum_{k=n-2}^{1} (a^{-3} b^{-1})^{k}$

$= b^{n} \dfrac{(a^{-3}b^{-1})^{n-2} - (a^{-3}b^{-1})^{2}}{1 - a^{-3}b^{-1}}$

CASE B

$y[n] = 0$

$$y[n] = b^n \frac{(a^{-3}b^{-1})^{n-2} - (a^{-3}b^{-1})^2}{1 - a^{-3}b^{-1}} u[n-3]$$

c) $x[n] = u[n] \quad h[n] = a^n(u[n] - u[n-100])$

$y[n] = u[n] * a^n u[n] - u[n] * a^n u[n-100]$

Use the given formula

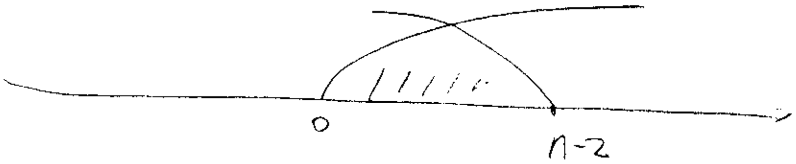
$a^{-2n} u[n] \quad x[n]$

$y[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k]$

$= b^n \sum_{k=-\infty}^{+\infty} \left(\frac{a}{b}\right)^k u[k] u[n-k-2]$

$u[k] = \begin{cases} 1 & k \geq 0 \\ 0 & k < 0 \end{cases}$

$u[n-k-2] = \begin{cases} 1 & n-k-2 \geq 0 & k < n-2 \\ 0 & & k > n-2 \end{cases}$



CASE A $\quad n-2 \geq 0$

$$y[n] = b^n \sum_{k=0}^{n-2} \left(\frac{a}{b}\right)^k$$

$$= b^n \frac{1-\left(\frac{a}{b}\right)^{n-1}}{1-\frac{a}{b}}$$

CASE B

$$y[n] = b^n \frac{1-\left(\frac{a}{b}\right)^{n-1}}{1-\frac{a}{b}} u[n-2]$$